Space-angle product

The space-angle product describes the distribution of light over space and direction. It applies to the diffraction of waveforms as well as ray propagation through an optical system. The space-angle product cannot be made smaller without discarding precious light; nor can it be arbitrarily increased without reduction in contrast. Ergo, effective management of the space-angle product is paramount within an optical system.

A waveform spreads laterally as it propagates. This lateral spread nature of light is effectively described by the space-angle product: [1]

$$A\Omega \geq \lambda^2,$$

in which A is the area of the spot, Ω is the solid angle of divergence, and λ is the wavelength. This product is directly related to the Heisenberg uncertainty principle. Its limit cannot be exceeded without creation of an evanescent wave which decays rapidly over distance. The one-dimensional version of the space-angle product is

$$\phi_0 \beta \geq \frac{4}{\pi}\lambda$$

$$\phi_0 \beta \geq 1.27\lambda$$

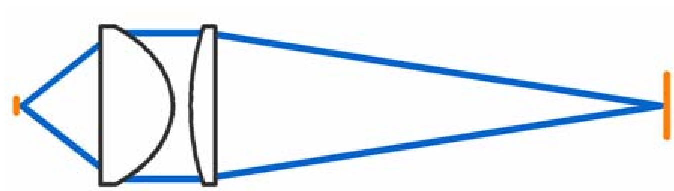
in which ϕ_0 is the diameter of a circular spatial profile, and β is the full-angle of divergence. In large-angle format, this becomes

$$\phi_0 (2\sin\alpha) \geq 1.27\lambda,$$

where α is the half-angle of divergence. The minimum space-angle product occurs for a Gaussian beam. The space-angle product is valid within any refractive media, as long as the wavelength is represented by its proper value within the refractive medium.

A ray pattern also obeys the space-angle product. The company logo for L-A-Omega is a perfect example of true scale below. The geometric areas A_n and solid angles Ω_n obey the space angle product as

$$A_0\Omega_0 = A_1\Omega_1 = \cdots = A_n\Omega_n \geq \lambda^2.$$



The space-angle product cannot be made smaller with discarding precious light. Furthermore, it cannot be arbitrarily increased without creation of optical aberrations that reduce contrast. The space-angle product must be skillfully managed for maximum collection of light.

[1] G.H. Seward, "Two-dimensional space-angle product of a Gaussian beam," *Opt. Eng.* 40(2):1959–1962 (2001).