Evanescent field

Under certain constraints, a sinusoidal plane wave may be transformed into a waveform with an *evanescent field*. By definition, an evanescent field is short-lived. It exists over a very short distance or time. The evanescent field is related to the temporal curvature as follows.

The temporal curvature of an electromagnetic wave $\ddot{\mathbf{E}}$ with steady amplitude over time is

$$\ddot{\mathbf{E}} = \frac{\partial^2}{\partial z^2}\mathbf{E} = -\omega^2\mathbf{E}.$$

The negative scalar component represents an opposing curvature that curves the electric field against its direction and towards zero. A positive scalar component represents a supporting spatial curvature that curves the electric field along its direction and away from zero. A negative spatial curvature creates a sinusoid, while a positive spatial curvature creates an exponential decay or growth.

The spatial curvature $\nabla^2\mathbf{E}$ may be expressed as

$$\nabla^2\mathbf{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E} = -\left(k_x^2 + k_y^2 + k_y^2\right)\mathbf{E},$$

where: k_x, k_y, and k_z are the spatial frequencies along x, y, and z; and $-k_x^2\mathbf{E}$, $-k_y^2\mathbf{E}$, and $-k_z^2\mathbf{E}$ represent the spatial curvatures along long x, y, and z.

The sum of the spatial curvatures are related to temporal curvature as

$$-\left(k_x^2 + k_y^2 + k_y^2\right)\mathbf{E} = -N^2\frac{\omega^2}{c^2}\mathbf{E}$$

If the refractive index is complex due to either dipole resonance or conduction, then the spatial curvature becomes a blend of opposing and supporting curvature in the form a sinusoid with exponential decay.

If the refractive index is purely real as in a dielectric, then the sum of the spatial curvatures becomes

$$-\left(k_x^2 + k_y^2 + k_y^2\right)\mathbf{E} = -N^2\frac{\omega^2}{c^2}\mathbf{E} = -k_N^2\mathbf{E},$$

where k_N represents the *ordinary spatial frequency* of the medium, and $-k_N^2\mathbf{E}$ represents the *ordinary spatial curvature* of the medium. If a spatial frequency component exceeds the ordinary spatial frequency, then a spatial curvature exceeds the ordinary spatial curvature in magnitude. Consequently, another spatial curvature must display a different polarity. Therefore, an electromagnetic wave within a dielectric can display both sinusoidal and exponential components if the ordinary spatial frequency is exceeded.

The ordinary spatial frequency is exceeding in several well-known structures. In an optical fiber, the spatial frequency along the border between the core and cladding is ordinary within the core, but extraordinary within the cladding. Consequently, the waveform resembles a sinusoid within the core, while it resembles an exponential decay in the cladding. The open structure of microwave window defines an extraordinary spatial frequency along the window, therefore the transmitted waveform is an evanescent field at normal to the window. At the *critical angle* of internal reflection, the spatial frequency along the interface becomes extraordinary for the external medium. Consequently, a plane wave of internal medium creates an evanescent field within the external medium.

The sum of the spatial curvatures must equal the temporal curvature as scaled by the refractive index of the medium. Ergo, a positive spatial curvature may coexist with a negative spatial curvature. This occurs when a spatial frequency component is extraordinary, or when the refractive index is complex due to conduction or resonance. Furthermore, a positive temporal curvature may coexist with a negative temporal curvature that occurs during emission.